Sequencing in the Run-Time Event Calculus

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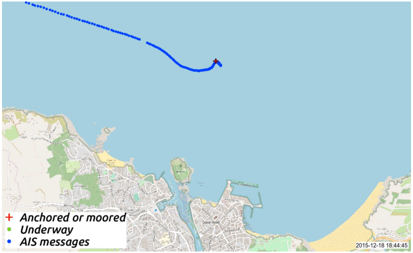
[3]University of Piraeus, Greece

Run-Time Event Calculus (RTEC)

- A composite event recognition framework that is formal, expressive and efficient.

$$
\begin{aligned}
&\mathbf{holdsFor}(\mathit{anchoredOrMoored}(\mathit{Vl}) = \text{true}, I) \leftarrow\\
&\quad \mathbf{holdsFor}(\mathit{stopped}(\mathit{Vl}) = \mathit{farFromPorts}, I_{sf}),\\
&\quad \mathbf{holdsFor}(\mathit{withinArea}(\mathit{Vl}, \mathit{anchorage}) = \text{true}, I_{a}),\\
&\quad \mathbf{intersect\_all}([I_{sf}, I_{a}], I_{sfa}),\\
&\quad \mathbf{holdsFor}(\mathit{stopped}(\mathit{Vl}) = \mathit{nearPorts}, I_{sn}),\\
&\quad \mathbf{union\_all}([I_{sfa}, I_{sn}], I).
\end{aligned}
$$

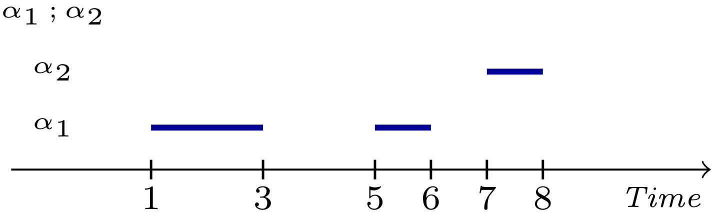


Artikis et al., An Event Calculus for Event Recognition. TKDE, 2015.
https://github.com/aartikis/rtec

Sequencing Operator for RTEC

Key Ingredients:

- Adjacency function: requires successive intervals.
- Composition function $\otimes$: constructs the output intervals.



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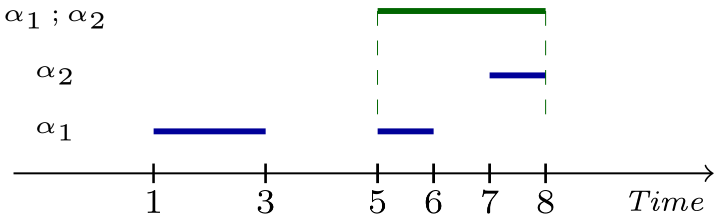

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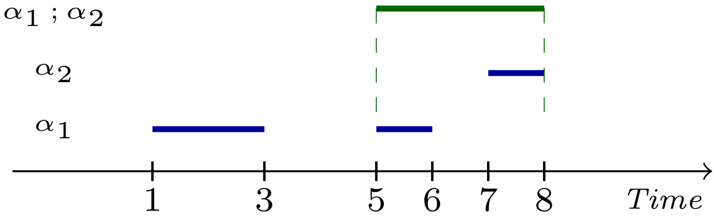
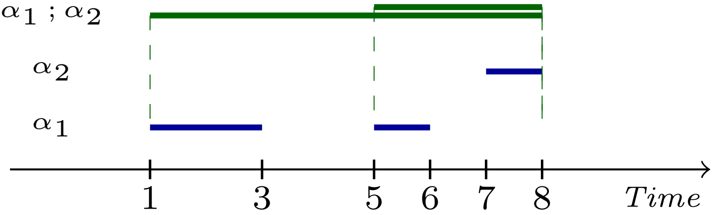


- Our sequencing operator produces maximal disjoint intervals.

Maximal Disjoint Interval Requirement

- Sequencing operators of complex event recognition frameworks do not meet the maximal disjoint interval requirement.



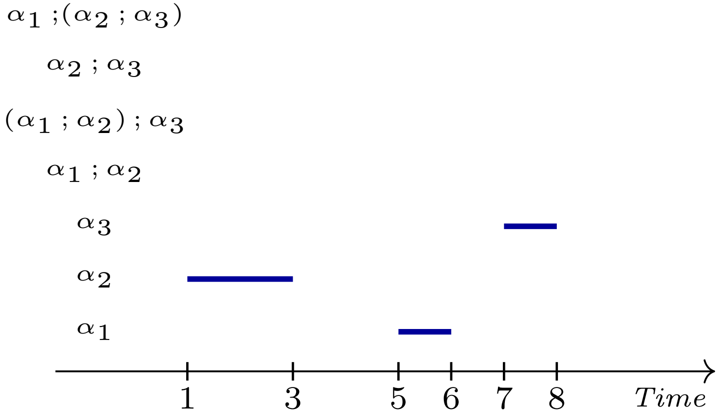
White W. M., Riedewald M., Gehrke J., Demers A. J.: What is “next” in event processing. PODS, 263–272. ACM, 2007.

Bucchi M., Grez A., Quintana A., Riveros C., Vansummeren S.: CORE: a complex event recognition engine. Proc. VLDB Endow., 15(9):1951–1964, 2022.

Alevizos E., Artikis A., Paliouras G.: Complex event recognition with symbolic register transducers. Proc. VLDB Endow., 17(11):3165–3177, 2024.

Associativity Requirement

- Associativity: required for optimising hierarchial patterns.



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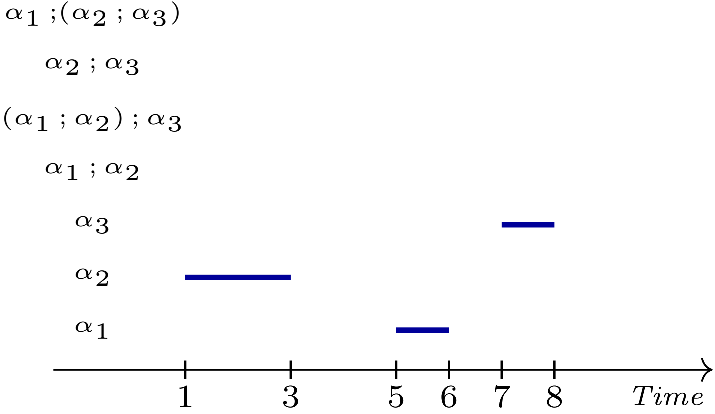
For a sequencing operator where $i_1 \otimes i_2 = i_2$:



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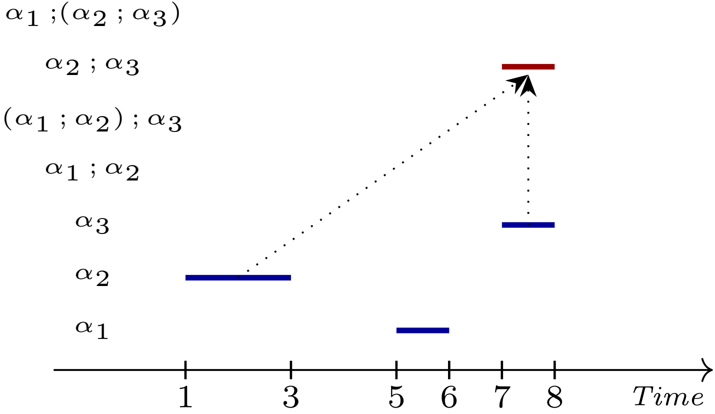

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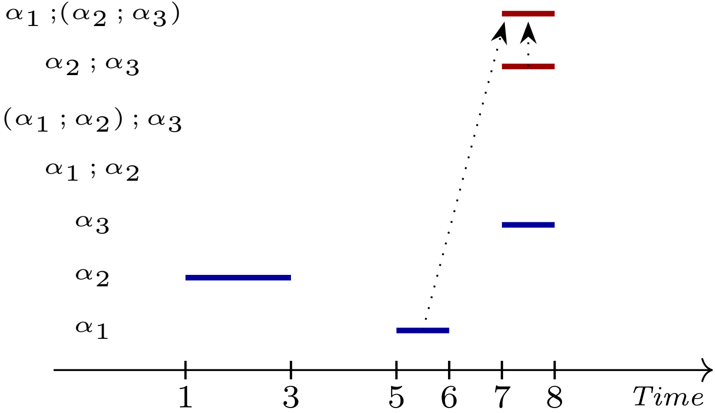

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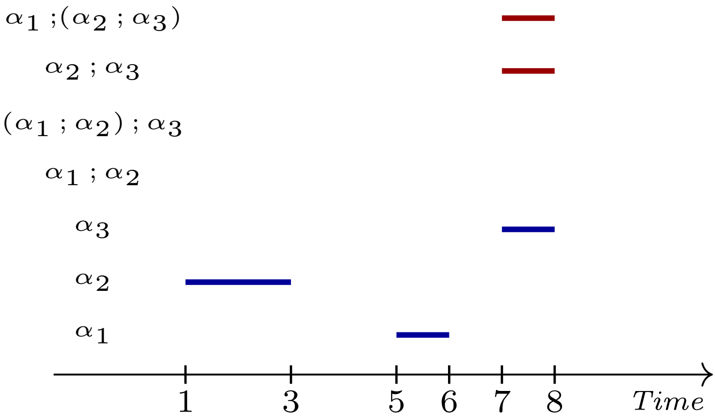

- Associativity is violated → Activities $(\alpha_1 ; \alpha_2) ; \alpha_3$ and $\alpha_1 ;(\alpha_2 ; \alpha_3)$ are assigned different lists of intervals.

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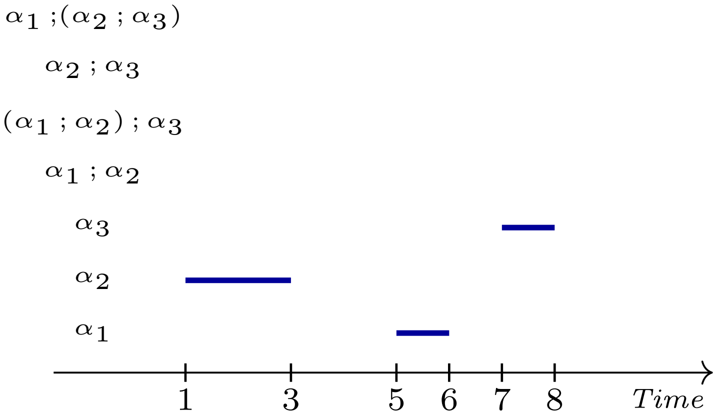

- Our sequencing operator satisfies associativity → Activities $(\alpha_1 ; \alpha_2) ; \alpha_3$ and $\alpha_1 ; (\alpha_2 ; \alpha_3)$ are assigned the same list of intervals.

RTEC$_S$: RTEC with Sequencing

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Pattern hierarchies in RTEC$_S$ are locally stratified logic programs.

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If I_1 and I_2 contain the maximal intervals of activities a_1 and a_2, then $\mathsf{seq}(I_1, I_2, I)$ computes the list of maximal intervals I of $a_1\,;a_2$.

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Complexity

RTEC_S evaluates $\mathsf{seq}(I_1, I_2, I)$ in time linear to the number of intervals in I_1 and I_2.

Experiments: Sequencing N Activities

Parameters		Reasoning Time (ms)		Computed Intervals	
N	D	RTEC$_S$	CORE	RTEC$_S$	CORE
3	10K	19	1K	500	148K
6	10K	23	7K	402	669K
12	10K	30	2K	35	109K
3	50K	82	109K	500	19M
6	50K	87	$>$600K	500	$>$30M
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- Pattern: $\alpha_1(Id)\,;\alpha_2(Id)\,;\ldots;\alpha_N(Id)$
  → A sequence of N different unary activities with the same argument.

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Experiments: Sequencing with Hierarchical Patterns

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3	31	39	2K	1.5K	1.5K	193K
6	63	84	18K	6.6K	6.6K	1.5M
12	240	400	48K	12.4K	12.4K	1.6M

- Patterns: $\alpha_1(Id)\,;\alpha_2(Id)\,;\ldots;\alpha_N(Id)$ and all its possible subpatterns, e.g.:
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  - etc.
- For $N = 12$, we have a hierarchy of 66 patterns.

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Experiments: Real Maritime Dataset

Window Size		Reasoning Time (sec)	Computed Intervals
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1	73K	2	18K
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4	272K	14	61K
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- Patterns: Composite Maritime Activities
- Dataset:
  - 18M position signals
  - 5K vessels
  - area near the port of Brest, France
  - 6 months of data

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Future Work:

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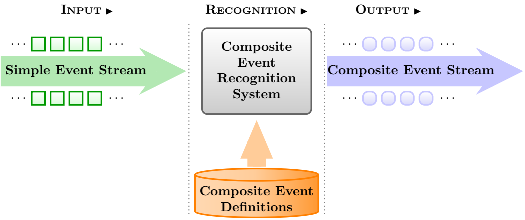
Appendix

Composite Event Recognition



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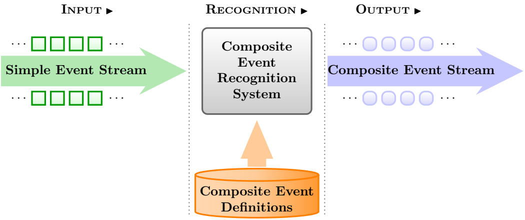

INPUT ▸

Simple Event Stream

RECOGNITION ▸

Composite Event Recognition System

Composite Event Definitions

OUTPUT ▸

Composite Event Stream

● Under way
★ Trawling
● AIS messages
● Other vessels
Fishing area
2016-02-02 02:02:29

https://cer.iit.demokritos.gr

(maritime situational awareness)

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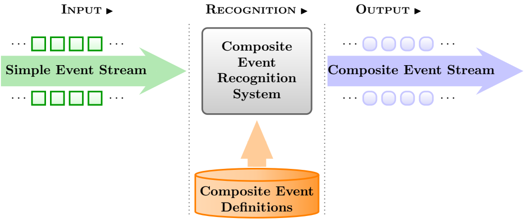

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(maritime situational awareness)

- Composite activity patterns often require sequencing.
- Example: phases of a fishing trip.

Event Calculus

- A logic programming language for representing and reasoning about events and their effects.
- Key components:
  - event (typically instantaneous).
  - fluent: a property that may have different values at different points in time.

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Run-Time Event Calculus (RTEC)

The Run-Time Event Calculus:

- Composite activity pattern matching over large-scale streams of events.

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Abstract Sequencing Model

An abstract sequencing model is a tuple $(T, \prec, \mathsf{S}, \otimes)$, where

- T is a set of time-stamps.

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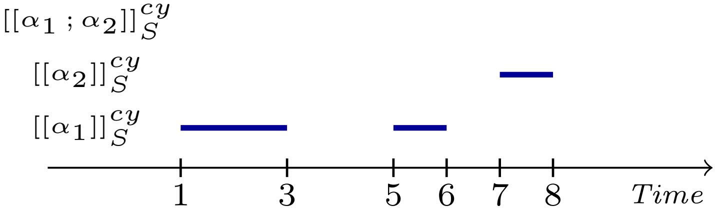

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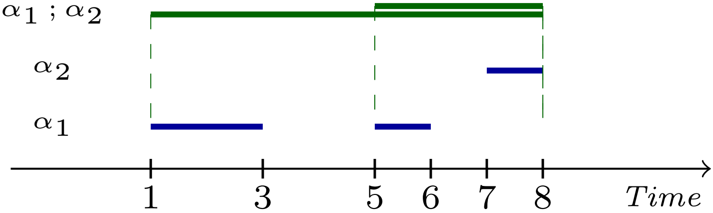

Sequencing: Requirements for RTEC

Requirement (Maximal Disjoint Intervals)

Consider a sequencing operator ";", a stream S and activities α_1 and α_2. $[[\alpha_1\, ;\, \alpha_2]]_S$ is composed of maximal disjoint intervals (MDIs).

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- These requirements imply compositionality.

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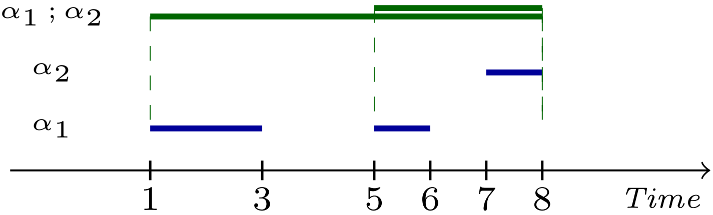

- Cayuga's sequencing operator violates the MDI requirement.

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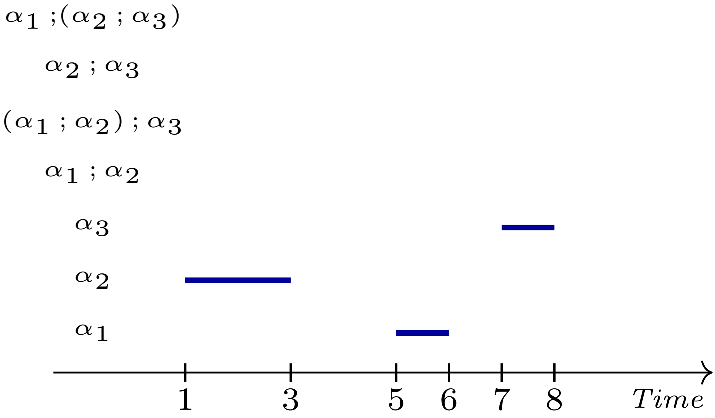

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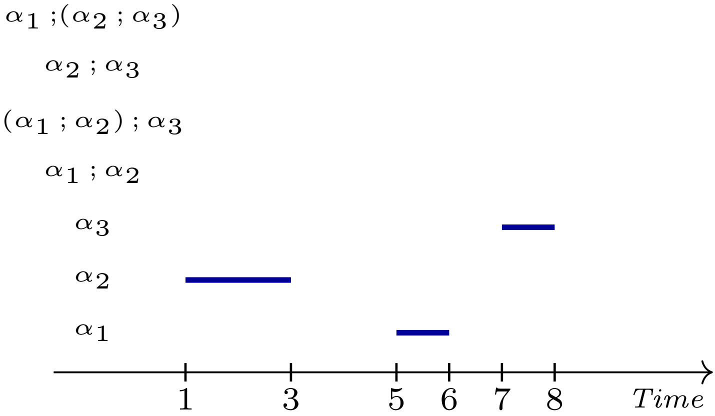

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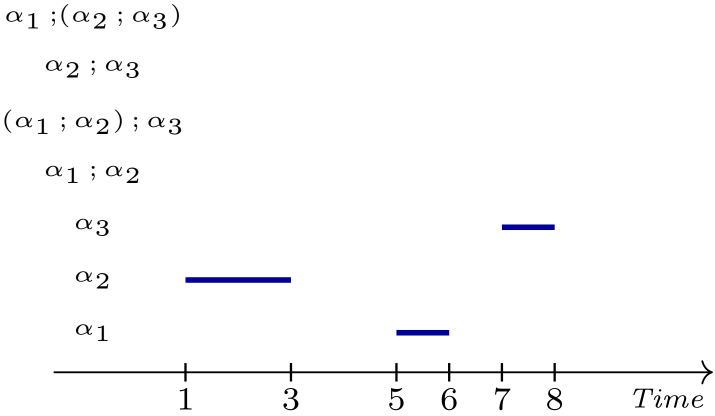

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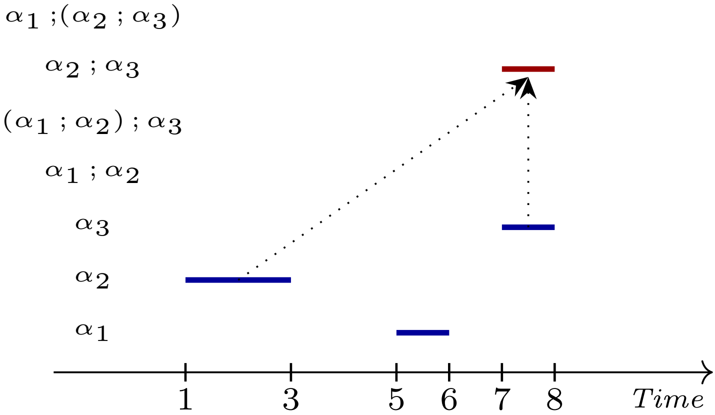

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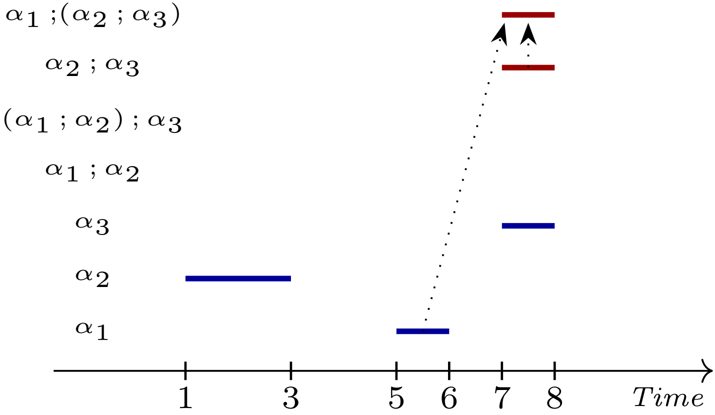

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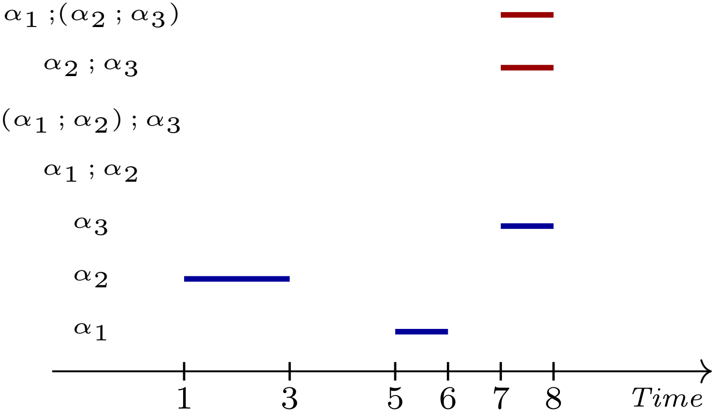

- Associativity fails: $[[(\alpha_1 ; \alpha_2) ; \alpha_3]]_S \neq [[\alpha_1 ;(\alpha_2 ; \alpha_3)]]_S$

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- Adjacency Mapping: $i_2 = \mathsf{A}(i_1, I_1, I_2)$ iff
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The sequencing operator of RTEC computes:

$$[[\alpha_1 \,;\alpha_2]]^{rt}_S = \{i_1 \otimes_{rt} i_2 \mid i_1 \in [[\alpha_1]]^{rt}_S \wedge i_2 = \mathsf{A}(i_1, [[\alpha_1]]^{rt}_S, [[\alpha_2]]^{rt}_S) \wedge i_2 \neq \emptyset\}$$

This sequencing operator is compatible with RTEC because:

- $[[\alpha_1 \,;\alpha_2]]^{rt}_S$ consists of maximal disjoint intervals.
- It is associative, i.e., $[[(\alpha_1 \,;\alpha_2) \,;\alpha_3]]^{rt}_S = [[\alpha_1 \,;(\alpha_2 \,;\alpha_3)]]^{rt}_S$, allowing optimised pattern hierarchies.