

Probabilistic Interval-based Event Recognition

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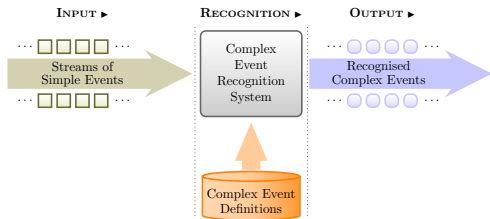
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<https://cer.iit.demokritos.gr>



Complex Event Recognition (Event Pattern Matching)^{*,†,‡}

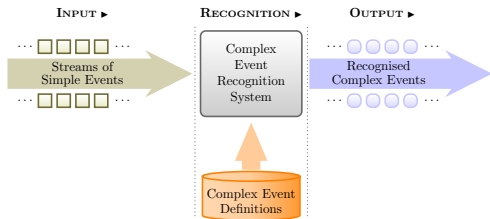


* Giatrakos et al, Complex event recognition in the Big Data era: A survey, VLDB Journal, 2020.

† Artikis et al, Dagstuhl Seminar on the Foundations of Composite Event Recognition. SIGMOD Record, 2020.

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Complex Event Recognition (Event Pattern Matching)*,†,‡



<https://cer.iit.demokritos.gr> (activity recognition)

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Event Calculus*

- A **logic programming language** for representing and reasoning about events and their effects.
- Key components:
 - **event** (typically instantaneous).
 - **fluent**: a property that may have different values at different points in time.

* Kowalski and Sergot, A Logic-based Calculus of Events. New Generation Computing, 1986.

Event Calculus*

- A **logic programming language** for representing and reasoning about events and their effects.
- Key components:
 - **event** (typically instantaneous).
 - **fluent**: a property that may have different values at different points in time.
- Built-in representation of **inertia**:
 - $F = V$ holds at a particular time-point if $F = V$ has been *initiated* by an event at some earlier time-point, and not *terminated* by another event in the meantime.

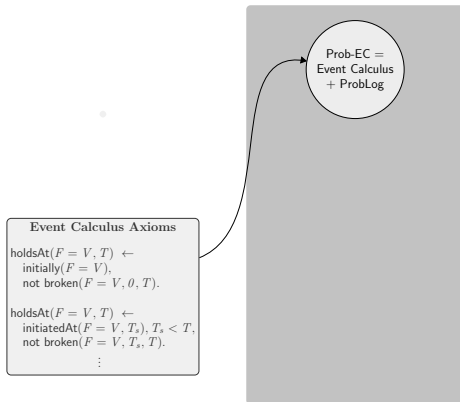
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Event Calculus + ProbLog

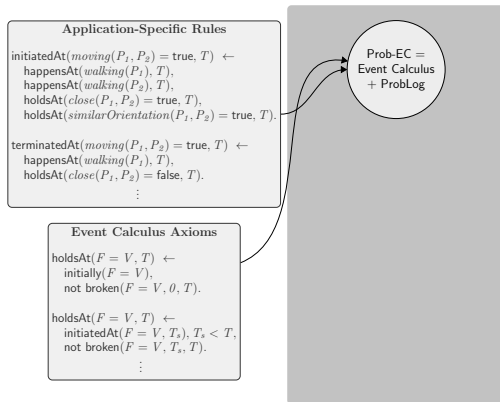


Prob-EC =
Event Calculus
+ ProbLog

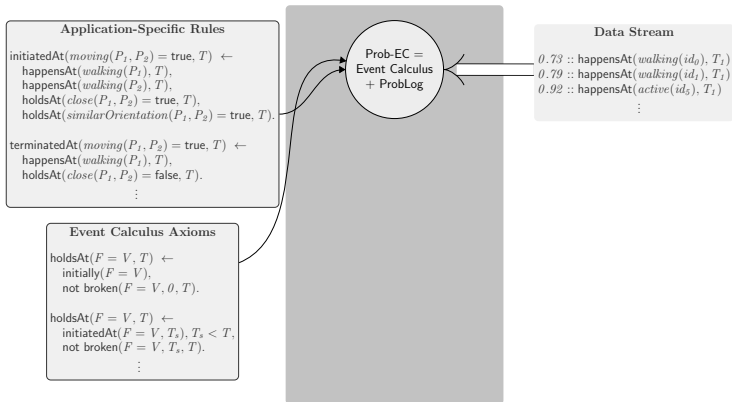
Event Calculus + ProbLog



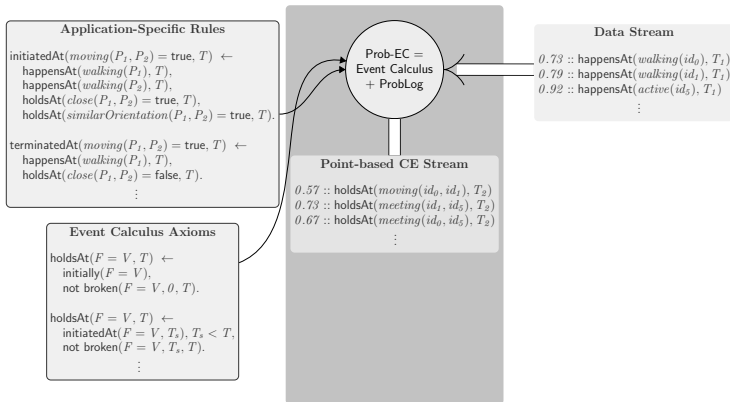
Event Calculus + ProbLog



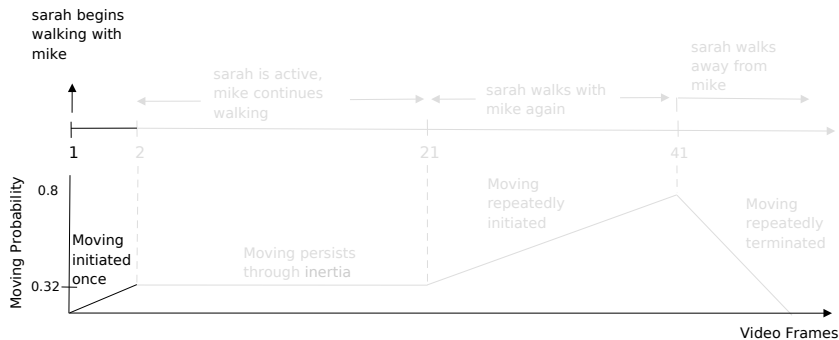
Event Calculus + ProbLog



Event Calculus + ProbLog



Instantaneous Probabilistic Recognition

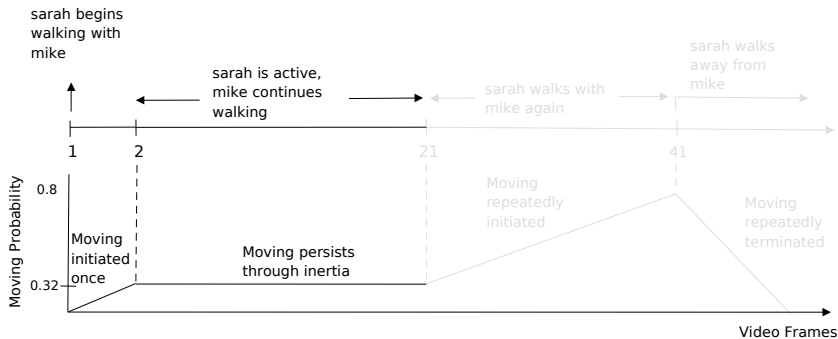


initiatedAt($moving(P_1, P_2) = true, T$) \leftarrow
happensAt($walking(P_1), T$),
happensAt($walking(P_2), T$),
holdsAt($close(P_1, P_2) = true, T$),
holdsAt($similarOrientation(P_1, P_2) = true, T$).

0.70 :: **happensAt**($walking(mike), 1$).
0.46 :: **happensAt**($walking(sarah), 1$).

terminatedAt($moving(P_1, P_2) = true, T$) \leftarrow
happensAt($walking(P_1), T$),
holdsAt($close(P_1, P_2) = false, T$).

Instantaneous Probabilistic Recognition

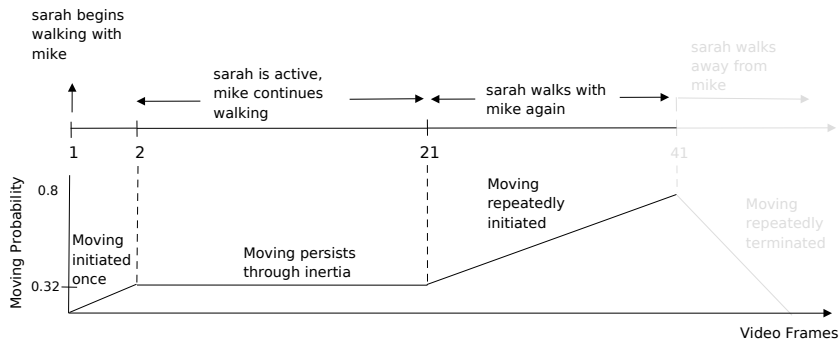


initiatedAt($moving(P_1, P_2) = true, T$) \leftarrow
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 0.73 :: **happensAt**($walking(mike), 2$).
 0.55 :: **happensAt**($active(sarah), 2$). ...

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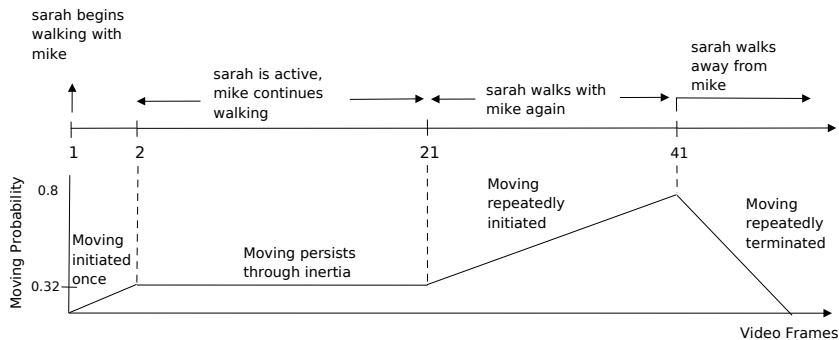


$\text{initiatedAt}(\text{moving}(P_1, P_2) = \text{true}, T) \leftarrow$
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 $0.55 :: \text{happensAt}(\text{active}(\text{sarah}), 2). \dots$
 $0.69 :: \text{happensAt}(\text{walking}(\text{mike}), 21).$
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Instantaneous Probabilistic Recognition

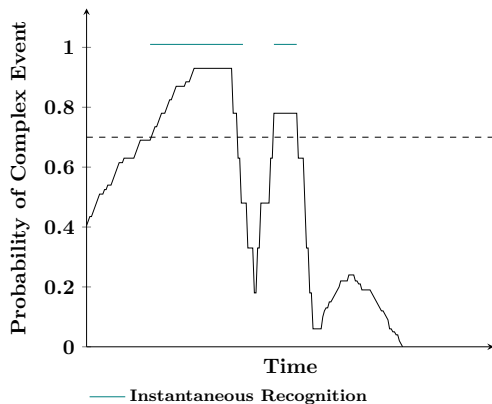


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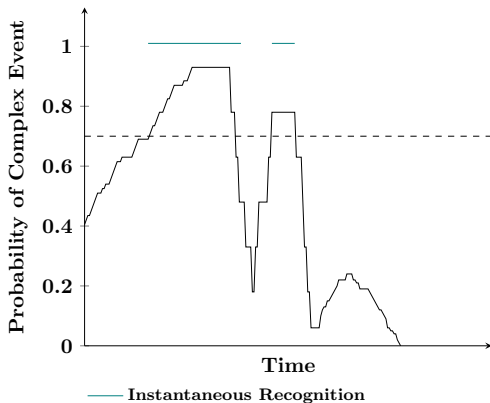
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 $0.82 :: \text{happensAt}(\text{inactive}(\text{mike}), 41).$
 $0.79 :: \text{happensAt}(\text{walking}(\text{sarah}), 41). \dots$

Instantaneous Recognition*



* Skarlatidis et al, A Probabilistic Logic Programming Event Calculus. Theory & Practice of Logic Programming, 2015.

Instantaneous Recognition*

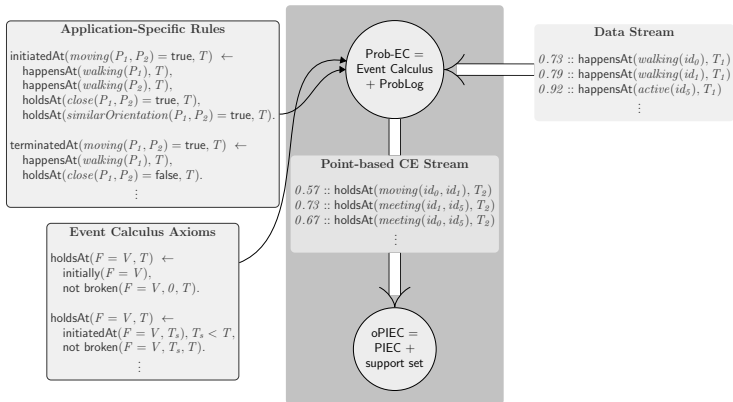


Higher accuracy than crisp reasoning in the presence of:

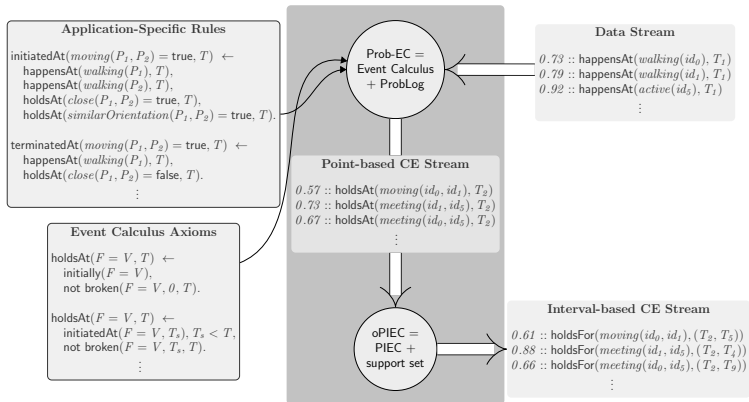
- several initiations and terminations;
- few probabilistic conjuncts.

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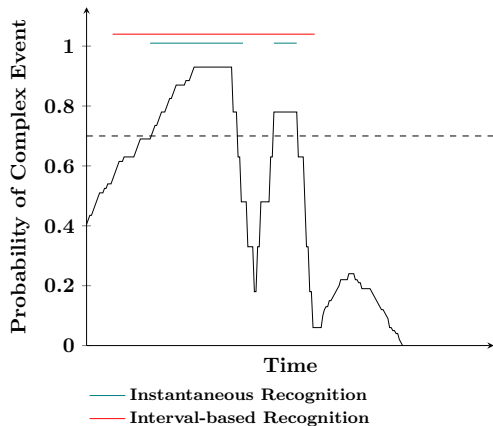
Online Probabilistic Interval-Based Event Calculus



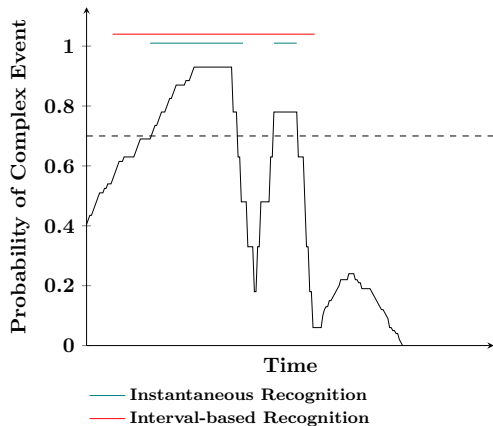
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Instantaneous vs Interval-based Recognition

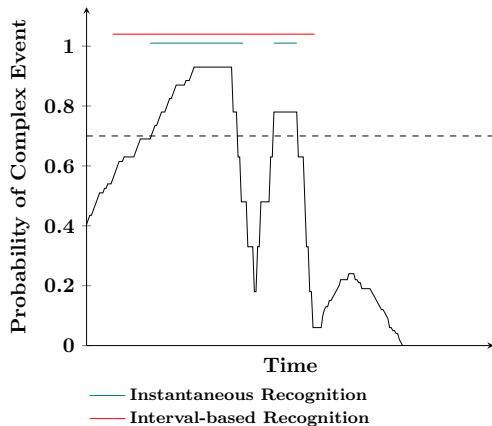


Instantaneous vs Interval-based Recognition



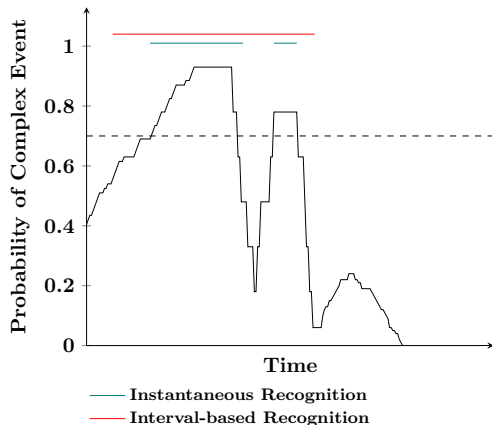
- **Interval Probability:** average probability of the time-points it contains.

Instantaneous vs Interval-based Recognition



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- **Probabilistic Maximal Interval:**
 - interval probability above a given threshold;
 - no super-interval with probability above the threshold.

Instantaneous vs Interval-based Recognition



- **Interval Probability:** average probability of the time-points it contains.
- **Probabilistic Maximal Interval:**
 - interval probability above a given threshold;
 - no super-interval with probability above the threshold.
- Probabilistic maximal interval computation via **maximal non-negative sum interval** computation.

Interval-based Recognition*

Interval Computation Correctness

An interval is computed iff it is a probabilistic maximal interval.

* Artikis et al, A Probabilistic Interval-based Event Calculus for Activity Recognition. Annals of Mathematics and Artificial Intelligence, 2020.

Interval-based Recognition*

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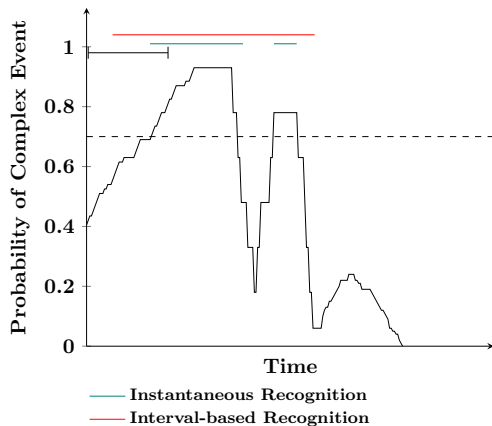
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Complexity

The computation of probabilistic maximal intervals is linear to the dataset size.

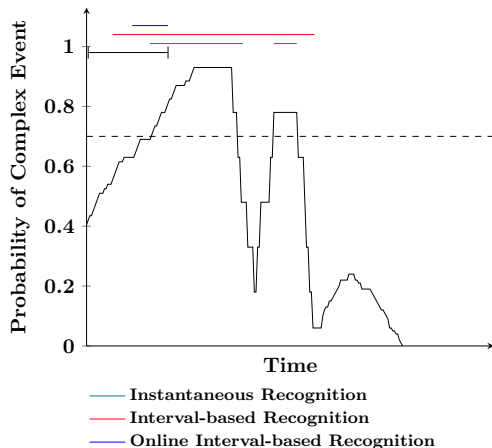
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Online Interval-based Recognition



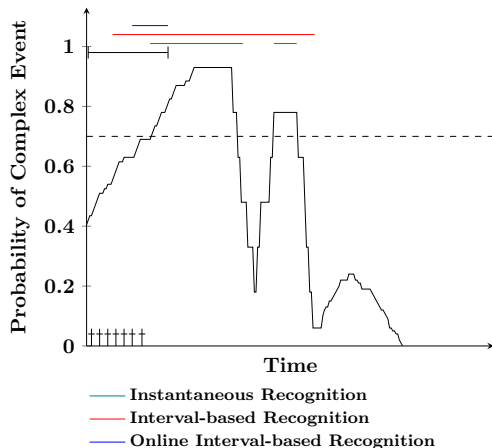
- Windowing.

Online Interval-based Recognition



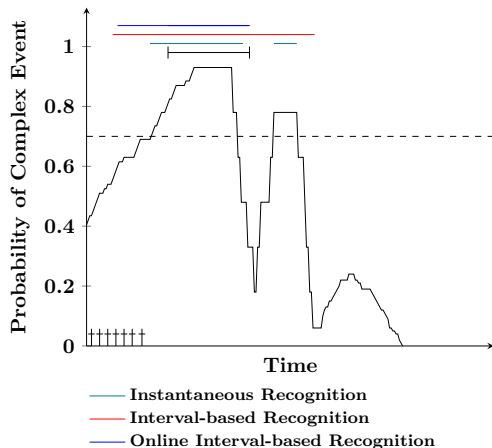
- Windowing.
- Probabilistic maximal interval computation.

Online Interval-based Recognition



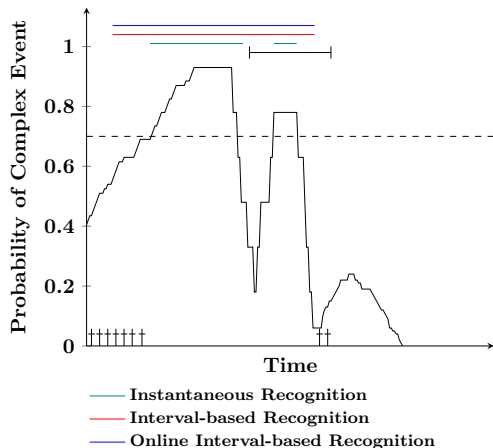
- Windowing.
- Probabilistic maximal interval computation.
- Caching **potential starting points**.
 - Caches time-point t iff the probability of an interval starting at t cannot be increased by extending it to the left.

Online Interval-based Recognition



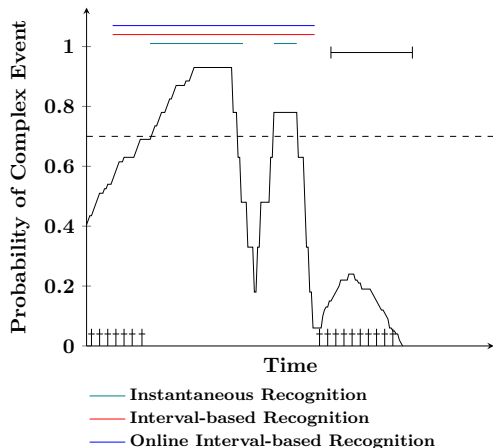
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Online Interval-based Recognition: Properties

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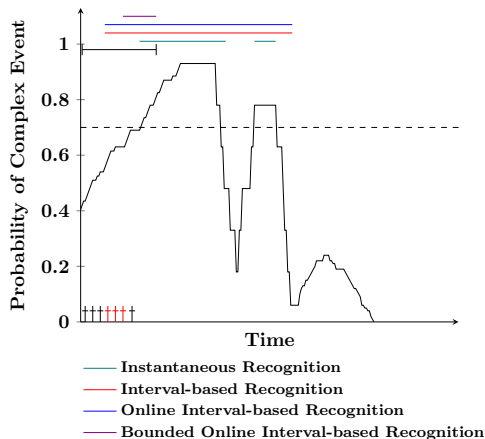
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The computation of probabilistic maximal intervals is linear to the window and memory size.

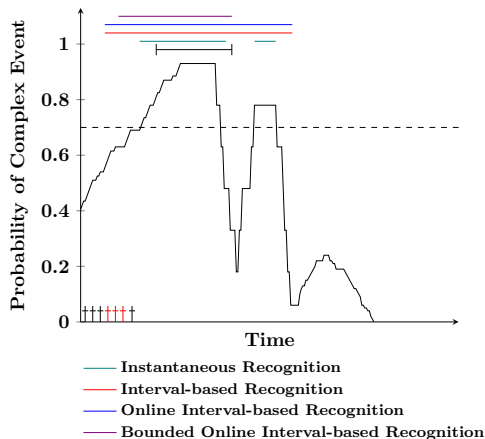
Bounded Online Interval-based Recognition*



- Complex event duration statistics favor more recent potential starting points.

* Mantenoglou et al, Online Probabilistic Interval-Based Event Calculus. ECAI, 2020.

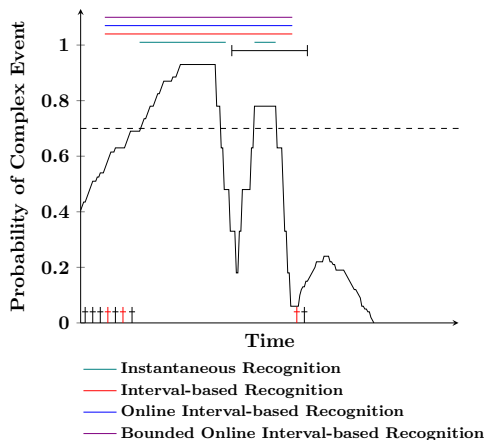
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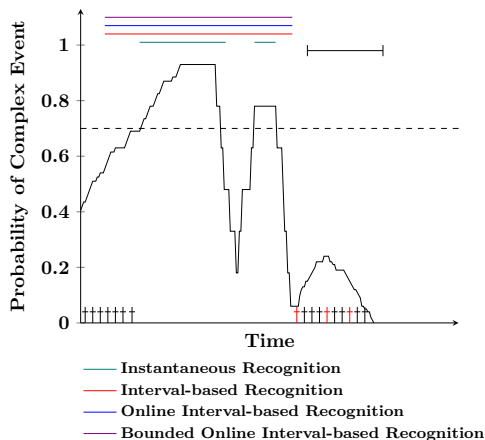
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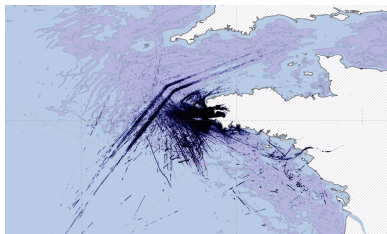
Bounded Online Interval-based Recognition*



- Complex event duration statistics favor more recent potential starting points.
- Comparable accuracy to batch reasoning.

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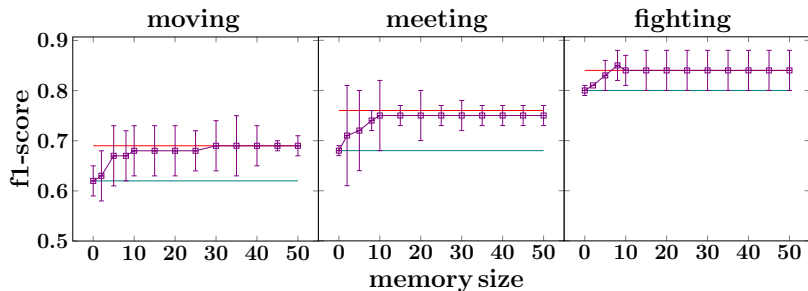
Experimental Setup



- Human Activity Recognition:
 - Input: manually annotated simple activities on individual video frames.
 - Output: maximal intervals of complex activities.
- Maritime Situational Awareness:
 - Input: vessel position signals from the area of Brest, France.
 - Output: maximal intervals of complex vessel activities.
- <https://github.com/Periklismant/oPIEC>

Experimental Results: Human Activity Recognition

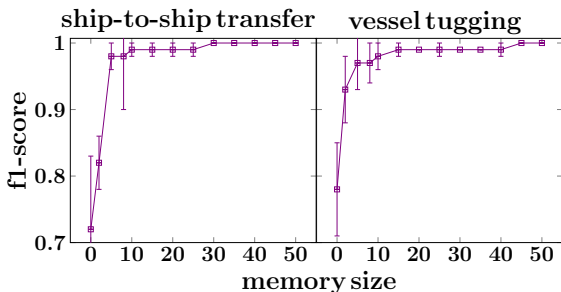
Comparison against ground truth



- Instantaneous Recognition
- Interval-based Recognition
- Bounded Online Interval-based Recognition

Experimental Results: Maritime Situational Awareness

Performance of bounded online recognition against batch recognition



Summary & Next Steps

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- Online reasoning over noisy streams.
- Optimal history compression for correct interval computation.
- Reproducible evaluation on benchmark and real data.

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Next Steps:

- Point-based probabilistic inference is required.
⇒ Interval-based EC for probabilistic, run-time reasoning.
- Symbolic 'simple events' are required.
⇒ Integration into a neuro-symbolic framework.
- Support uncertainty in complex event definitions.

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Resources

<https://github.com/periklismant/oPIEC>
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Appendix

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	<i>0</i>	<i>0.5</i>	<i>0.7</i>	<i>0.9</i>	<i>0.4</i>	<i>0.1</i>	<i>0</i>	<i>0</i>	<i>0.5</i>	<i>1</i>

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>ln</i>	<i>0</i>	<i>0.5</i>	<i>0.7</i>	<i>0.9</i>	<i>0.4</i>	<i>0.1</i>	<i>0</i>	<i>0</i>	<i>0.5</i>	<i>1</i>
<i>L</i>	<i>-0.5</i>	<i>0</i>	<i>0.2</i>	<i>0.4</i>	<i>-0.1</i>	<i>-0.4</i>	<i>-0.5</i>	<i>-0.5</i>	<i>0</i>	<i>0.5</i>

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
l_n	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
L	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5

$$\sum_{i=s}^e L[i] \geq 0 \Leftrightarrow P([s, e]) \geq \mathcal{T}$$

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
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<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>										-0.9

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<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>									-0.9	-0.9

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<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>								-0.9	-0.9	-0.9

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<i>dp</i>							-0.9	-0.9	-0.9	-0.9

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<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval-based Recognition

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<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[s, e] = \begin{cases} dp[e] - prefix[s-1] & \text{if } s > 1 \\ dp[e] & \text{if } s = 1 \end{cases}$$

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[s, e] = \begin{cases} dp[e] - prefix[s-1] & \text{if } s > 1 \\ dp[e] & \text{if } s = 1 \end{cases}$$

$$dprange[s, e] \geq 0 \Rightarrow \exists e^* : e^* \geq e, P([s, e^*] \geq \mathcal{T})$$

Interval-based Recognition

	$\uparrow\downarrow$									
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval-based Recognition

	$\uparrow\downarrow$									
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 1] = dp[1] = 0.1 \geq 0$$

Interval-based Recognition

	\uparrow	\downarrow								
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval-based Recognition

	↑	↓								
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 2] = dp[2] = 0.1 \geq 0$$

Interval-based Recognition

	\uparrow		\downarrow							
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 3] = dp[3] = 0.1 \geq 0$$

Interval-based Recognition

	\uparrow			\downarrow						
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 4] = dp[4] = 0.1 \geq 0$$

Interval-based Recognition

	\uparrow				\downarrow					
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 5] = dp[5] = 0 \geq 0$$

Interval-based Recognition

	\uparrow					\downarrow				
Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 6] = dp[6] = -0.4 < 0$$

Interval-based Recognition

Time	\uparrow 1	2	3	4	5	\downarrow 6	7	8	9	10
<i>ln</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[1, 6] = dp[6] = -0.4 < 0$$

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>ln</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9


$$dprange[2, 6] = dp[6] - prefix[1] = 0.1 \geq 0$$

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[2, 7] = dp[7] - prefix[1] = -0.4 < 0$$

Interval-based Recognition



Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

$$dprange[2, 7] = dp[7] - prefix[1] = -0.4 < 0$$

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval Computation Correctness

An interval is computed iff it is a probabilistic maximal interval.

Interval-based Recognition

Time	1	2	3	4	5	6	7	8	9	10
<i>In</i>	0	0.5	0.7	0.9	0.4	0.1	0	0	0.5	1
<i>L</i>	-0.5	0	0.2	0.4	-0.1	-0.4	-0.5	-0.5	0	0.5
<i>prefix</i>	-0.5	-0.5	-0.3	0.1	0	-0.4	-0.9	-1.4	-1.4	-0.9
<i>dp</i>	0.1	0.1	0.1	0.1	0	-0.4	-0.9	-0.9	-0.9	-0.9

Interval Computation Correctness

An interval is computed iff it is a probabilistic maximal interval.

Complexity

The computation of probabilistic maximal intervals is linear to the dataset size.